Backward Angle Fermion-Boson Scattering*

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In a Regge-type treatment of Fermion-boson reactions, backward angle differential cross sections are dominated by complex conjugate pairs of u-channel Regge trajectories (this is true for spins $\frac{1}{2}$ and 0 or 1, and probably in general). It is demonstrated here that generally, these differential cross sections and associated final-state polarizations will not exhibit oscillatory behavior as a function of u and s, in spite of interferences between the poles of such complex conjugate pairs. This result is in agreement with previous findings in the special case of π-N scattering.

I. INTRODUCTION

It has been pointed out that at backward angles in pion-nucleon scattering, for which the kinematic invariant u is <0, the Regge trajectory amplitudes exhibit complex conjugate trajectories, which, however, interfere with each other in such a way that no oscillatory terms appear in the backward angle differential cross section.[1–3] The question remained whether oscillatory behavior would occur in boson-Fermion reactions with higher spin. Examples of such processes would be vector-spinor scattering such as elastic proton-deuteron scattering, or inelastic processes such as pion photoproduction from nucleons. One might, at first sight, expect oscillatory terms due to interference between conjugate trajectories α_1 and $\alpha_2=\alpha_1^*$, having the asymptotic form

$$\mathrm{Re}(s/s_0)^{\alpha_1+\alpha_2^*}=S^{2\,\mathrm{Re}\alpha_1}\times\cos[2\,\mathrm{Im}\alpha_1\ln(s/s_0)].$$

Observation of such oscillations would provide a striking confirmation of the Regge-pole hypothesis. This paper, however, demonstrates that, with regard to the leading asymptotic behavior in s, no oscillatory cross sections will occur. It is shown, too, that in the same asymptotic limit, the polarization of a final-state particle will not be oscillatory in the case of an unpolarized initial state. Only when the initial state is polarized (experimentally difficult to achieve), or when two close-lying pairs of trajectories occur,[4] might oscillations be observed.

II. GENERAL PROPERTIES OF THE AMPLITUDES

Let us begin with a brief review of several features of $\pi-N$ scattering amplitudes. Let A, B be the conventional scalar amplitudes, and q_1, q_2 the initial and final pion momenta in the cm frame. The T matrix will then be $A+B(\mathbf{q}_1+\mathbf{q}_2)/2$. Let F_{l^+}, F_{l^-} denote definite parity partial-wave amplitudes, for final states with $J=l\pm\frac{1}{2}$. These F's exhibit the MacDowell symmetry[5] (here written for u-channel amplitudes since we are to be interested in backward angles of the s channel where u-channel Regge poles dominate),

$$F_{l^+}(u^{1/2})=-F_{(l+1)^-}(-u^{1/2}).\qquad(1)$$

Also, $F_{l^+}(u^{1/2})+F_{(l+1)^-}(u^{1/2})$ is regular at $u=0$, while $F_{l^+}(u^{1/2})-F_{(l+1)^-}(u^{1/2})$ has a $u^{1/2}$ or $1/(u)^{1/2}$ singularity. From this, it follows that the J-plane singularities of these partial-wave amplitudes are complex conjugate poles with complex conjugate residues for $u<0$.

Incidentally, a dispersion-theoretic treatment of this problem by Amati *et al.*[6] has confirmed these properties.

A more general situation will now be considered: Let a, c represent initial and final bosons with respective spins s_a, s_c; let b, d represent initial and final fermions with respective spins s_b, s_d. One can express two particular helicity amplitudes[7] in the αth channel ($\alpha=s$ or u here) as follows:

$$f_{\lambda c\lambda d;\,\lambda a\lambda b}(\theta,\phi)=\sum_J\,(J+\tfrac{1}{2})a_J'(W)\times e^{i(\lambda-\mu)\phi}d_{\lambda,\mu}{}^J(\theta),$$
$$f_{-\lambda c,-\lambda d;\,\lambda a\lambda b}(\theta,\phi)=\sum_J\,(J+\tfrac{1}{2})b_J'(W)\times e^{i(\lambda+\mu)\phi}d_{\lambda,-\mu}{}^J(\theta),\qquad(2)$$

where $\lambda=\lambda a-\lambda b$,
$\mu=\lambda c-\lambda d$,
$W=$ total c.m. energy in αth channel.

Let

$$\beta_{1,2}'^J(W)=a_J'(W)\pm b_J'(W).\qquad(3)$$

Then $\beta_{1,2}'^J$ are recognized to be definite parity partial-wave amplitudes.

In a recent study, Marx[8] has established that for spin 0 or 1 bosons, these β's have the following two properties. Of $\beta_1\pm\beta_2$, one combination has a $u^{1/2}$ behavior, while the other is regular at $u=0$. This establishes the same J-plane behavior for these β's as in the $\pi-N$ case. Also, one has that $\beta_1{}^J(+W)=\eta\beta_2{}^J(-W)$, where $\eta=\pm1$, and in both these statements, the sign depends on the particular set of helic-

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ities. These results can be obtained by explicit evaluation of T-matrix elements, in terms of scalar amplitudes multiplied by kinematic invariants, between actual helicity eigenfunctions of the relevant particles, and subsequent comparison with Eqs. (2). One may plausibly conjecture this to be true for all boson and Fermion spins.[9] The above being true, one then has the possibility of oscillations from interferences between complex conjugate poles. This latter situation now will be shown not to occur.

Let us now consider some properties of the $d_{\lambda\mu}{}^J$. One can write[8]

$$d_{\lambda\mu}{}^J(\theta)=(1+x)^{-(\lambda+\mu/2)}(1-x)^{-(\lambda-\mu/2)}\sum_{k=J-\lambda}^{J+\lambda}A_kP_k(x)\,, \quad (4)$$

where $x=\cos\theta$ and the exact form of A_k will not be important here. Using $d_{\lambda,-\mu}{}^J(x)=(-)^{J+\lambda}d_{\lambda\mu}{}^J(-x)$, one obtains

$$d_{\lambda,-\mu}{}^J(x)=(1-x)^{-(\lambda+\mu/2)}(1+x)^{-(\lambda-\mu/2)}\times\sum_{k=J-\lambda}^{J+\lambda}A_kP_k(-x)x(-)^{J+\lambda}. \quad (4a)$$

Anticipating Reggeization of the amplitudes (2) by means of a Watson-Sommerfeld transformation, one can rewrite Eq. (2) keeping only the highest order polynomial in the expansions of $d_{\lambda\mu}{}^J$, and redefining the β's as in Eq. (5) for simplicity:

$$\begin{aligned}a'^JA_{J+\lambda}&\equiv a^J=(\beta_1{}^J+\beta_2{}^J)/2\,,\\ b'^JA_{J+\lambda}&\equiv b^J=(\beta_1{}^J-\beta_2{}^J)/2;\end{aligned} \quad (5)$$

$$\begin{aligned}f_{\lambda c\lambda d;\,\lambda a\lambda b}(\theta,\phi=0)&\approx\sum_J(J+\tfrac{1}{2})(\beta_1{}^J(W)+\beta_2{}^J(W))\\&\times\left(\sin\frac{\theta}{2}\Big/\cos\frac{\theta}{2}\right)^{\mu}P_{J+\lambda}(x)\left(2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)^{-\lambda},\\ f_{-\lambda c,-\lambda d;\,\lambda a\lambda b}(\theta,\phi=0)&\approx\sum_J(J+\tfrac{1}{2})(\beta_1{}^J(W)-\beta_2{}^J(W))\\&\times\left(\cos\frac{\theta}{2}\Big/\sin\frac{\theta}{2}\right)^{\mu}P_{J+\lambda}(x)\left(2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)^{-\lambda}.\end{aligned} \quad (6)$$

III. REGGEIZATION AND ANALYTIC CONTINUATION

Performing the Watson-Sommerfeld[10] transformation as in Ref. 10, one sees that no difficulties are introduced by the redefinition (5); the β's still have the analyticity in u, j discussed above. The use of P_J's rather than the functions suggested by Gell-Mann *et al.*,[11] will also not affect the essential structure of this argument. The modifications of Ref. 11 affect singularities in the left-hand J plane and are thus not crucial when the Watson-Sommerfeld contour is taken at $\mathrm{Re}J\geq-\frac{1}{2}$.

Considering now u-channel amplitudes, for $u<0$, one obtains

$$\begin{aligned}f_{\lambda c\lambda d,\,\lambda a\lambda b}&=\left(\sin\frac{\theta u}{2}\Big/\cos\frac{\theta u}{2}\right)^{\mu}\\&\times(\sin\theta u)^{-\lambda}[\rho\zeta_\alpha+\eta\rho^*\zeta_{\alpha^*}]\,,\end{aligned} \quad (7)$$

$$\begin{aligned}f_{-\lambda c,-\lambda d,\,\lambda a\lambda b}&=\left(\cos\frac{\theta u}{2}\Big/\sin\frac{\theta u}{2}\right)^{\mu}\\&\times(\sin\theta u)^{-\lambda}[\rho\zeta_\alpha-\eta\rho^*\zeta_{\alpha^*}]\,,\end{aligned} \quad (8)$$

where $\zeta_\alpha=(1+\epsilon e^{-i\pi\alpha})P_{\alpha+\lambda}(-\cos\theta u)/(2\cos\pi\alpha)$, $\rho=$ residue of $(J+\frac{1}{2})\beta$ at pole $J=\alpha\times$miscellaneous "irrelevant" factors, $\epsilon=$signature of trajectory. Here, the physical trajectory was considered to have the same parity as the β_1 amplitude rather than the β_2 amplitude, but this choice is not essential to the main argument.

In order to obtain an expression for the s-channel differential cross section, one must be able to relate s-channel and u-channel amplitudes. In this regard, recent work by Trueman and Wick[12] proves most valuable; the chief result of relevance here is stated below for s-u channel crossing.

Let $f_{u\lambda c\lambda d;\,\lambda a\lambda b}{}^c$ be the analytic continuation of $fu_{\lambda c\lambda d;\,\lambda a\lambda b}$ to the s-channel physical region, and let $fs_{\alpha\beta\gamma\delta}$ be an s-channel helicity amplitude. Then

$$\begin{aligned}W_uf^c{}_{\lambda c\lambda d;\,\lambda a\lambda b}&=W_s\sum_{\alpha\beta\gamma\delta}(-)^nd_{\alpha\lambda d}{}^{Sd}(Xd)d_{\beta\lambda b}{}^{Sb}(Xb)\\&\times d_{\gamma\lambda c}{}^{Sc}(\psi c)d_{\delta\lambda a}{}^{Sa}(\psi a)\times fs_{\alpha\beta\gamma\delta}\,,\end{aligned} \quad (9)$$

where Xd, Xb, ψc, ψa are certain real angles defined in Ref. 12. From the orthogonality properties of the $d_{\lambda\mu}{}^J$, it immediately follows that

$$\begin{aligned}\left.\frac{d\sigma}{d\Omega}\right)_s&=\sum_{\alpha\beta\gamma\delta}|fs_{\alpha\beta\gamma\delta}|^2\\&=\sum_{\lambda c\lambda d\lambda a\lambda b}|f_u{}^c|^2\frac{|u|}{s}\,.\end{aligned} \quad (10)$$

The elegance of this result lies in the absence of cross terms. It should be emphasized that this approach makes it unnecessary to express all amplitudes in terms of scalar amplitudes and then to explicitly perform crossing in a lengthy and tedious calculation.

[9] Indeed, the MacDowell symmetry seems related to the fact that the intrinsic parities of a Fermion and anti-Fermion are opposite. Consider a Born term with intermediate s-channel Fermion state, in the Feynman-Dyson perturbation sense. It is equivalent to two different time-ordered graphs in the ordinary perturbation theory, with intermediate fermion and antifermion states, respectively, and consequently contributing to opposite parity partial waves. In a simple case such as πN, one can directly obtain the result that these two graphs then manifestly possess the MacDowell symmetry.

[10] T. W. B. Kibble, Phys. Rev. **131**, 2282 (1963).

[11] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen (to be published).

[12] T. L. Trueman and G. C. Wick, Brookhaven National Laboratory Report BNL 7301, 1963 (unpublished).

Now consider the analytic continuation f^c. In a case of elastic scattering, let μ=boson mass, m=Fermion mass. First note that as $s\to\infty$, one has generally[12] $\cos\theta u \sim -us/S^2$, where

$$S^2=[u-(m+\mu)^2][u-(m-\mu)^2].$$

Also,

$$\sin(\theta u/2)=(-ut)^{1/2}/s\,,$$
$$\cos(\theta u/2)=[(m^2-\mu^2)-us]^{1/2}/S\,. \tag{11}$$

The analytic continuation of $\sin(\theta u/2)$ is $\pm i(ut)^{1/2}/S$ when continuing to the region u, $t<0$; again, it is not necessary to consider the proper choice of sign in this argument. Similarly,

$$\cos(\theta u/2)\sim(-us)^{1/2}/S\equiv(+ut)^{1/2}/S\,.$$

Thus, $(\cos\theta u/2/\sin\theta u/2)$ has modulus 1 as $s\to\infty$, u fixed. Obviously, this behavior is also true for the case of all unequal masses, so long as $s\gg$(square of any mass involved).

Now one can most conveniently evaluate the right-hand side of Eq. (10) by considering pairs of amplitudes $f u_{\pm\{\lambda_f\},\{\lambda_i\}}{}^c$. From Eqs. (7), the sum of the squares of the moduli of two such amplitudes is

$$|\sin\theta u|^{-2\lambda}(2|\rho\zeta_\alpha|^2+2|\rho^*\zeta_{\alpha^*}|^2)\,. \tag{12}$$

Each amplitude separately contributes an oscillatory interference term $\mathrm{Re}(\rho^{*2}(\zeta_\alpha)^*(\zeta_{\alpha^*}))$ involving trigonometric functions whose argument is $2\,\mathrm{Im}(\alpha(u))\times\ln(s/s_0)$. However, the sum of the contributions from such a pair of amplitudes, and consequently the differential cross section, contains no oscillatory term, asymptotically in s.[13]

IV. POLARIZATION

Two types of measurements of final-state polarizations are of interest.[14] The simplest quantity to observe is the polarization of one of the final-state particles, denoted by $\langle S^{c,d}\rangle$, which for an unpolarized initial state, will be nonvanishing only in the direction perpendicular to the reaction plane when the reaction conserves parity.[15] Another quantity of interest is the correlation coefficient

$$C(lm)=\langle S_l{}^c S_m{}^d\rangle\,. \tag{13}$$

It is again convenient to employ helicity amplitudes when explicitly expressing these quantities. The resultant expressions for either of the above quantities, when the initial state is unpolarized, will be the sum of two or more terms, each of which has the form

$$\sum_{\lambda a\lambda b,\lambda c\lambda d} f_{\lambda c+a,\lambda d+b;\ \lambda a\lambda b}{}^* f_{\lambda c+a',\lambda d+b';\ \lambda a\lambda b}\, x_{\lambda c\lambda d aba'b'} \tag{14}$$

where $x\cdots$ will be a product, in general, of matrix elements of spin operators. As an example, one finds that if parity is conserved,

$$\langle S_1{}^c\rangle=\text{sum of two terms as in (14)}$$
$$=\sum_{\lambda c\lambda d,\lambda a\lambda b}\mathrm{Im}(f_{\lambda d\lambda c+1,\lambda a\lambda b}{}^* f_{\lambda d\lambda c;\ \lambda a\lambda b})$$
$$\times[(S_c-\lambda c)(S_c+\lambda c+1)]^{1/2}\,. \tag{15}$$

Employing the analytic continuation of helicity amplitudes of Trueman and Wick, Eq. (14) becomes

$$\sum_{\alpha\beta,\gamma\delta\mu\nu} f_{\gamma\delta;\ \alpha\beta}{}^{c*} f_{\mu\nu;\ \alpha\beta}{}^c\, Y_{\gamma\delta\mu\nu}\,, \tag{16}$$

where $Y\cdots$ will be equal to $x\cdots$ multiplied by rotation matrices of the angles involved in the analytic continuation, as defined in Ref. 12. The important feature of this expression is that the α and β indices are not involved in any contracted products with rotation matrices, this having resulted from the orthogonality properties of the rotation matrices involved here.

The following properties of helicity amplitudes should be noted:

$$f_{\lambda c\lambda d;\ \lambda a\lambda b}(\theta,\phi=0)=\sum_J (J+\tfrac{1}{2})\,{}_{\lambda c\lambda d}S_{\lambda a\lambda b}{}^J d_{\lambda\mu}{}^J(\theta)$$
$$=\sum_J (J+\tfrac{1}{2})\,{}_{\lambda a\lambda b}S_{\lambda c\lambda d}{}^J d_{\mu\lambda}{}^J(\theta)(-)^{(\lambda-\mu)}$$
$$=(-)^{\lambda-\mu} f_{\lambda a\lambda b;\ \lambda c\lambda d}(\theta,\phi=0)\,. \tag{17}$$

Using this relation in (14), one now obtains Eq. (16) in a slightly different form:

$$\sum_{\alpha\beta,\gamma\delta\mu\nu} f_{\alpha\beta;\ \gamma\delta}{}^{*c} f_{\alpha\beta;\ \mu\nu}{}^c\, Y_{\gamma\delta\mu\nu}\,, \tag{18}$$

where a phase independent of α, β has been absorbed into the $Y\cdots$. Now, one can employ Eq. (7) and again consider pairs of terms $(\alpha\beta)$ and $(-\alpha-\beta)$.

Equation (18) now will be the sum of terms as in (19):

$$Y_{\gamma\delta\mu\nu}([\rho\zeta_\alpha+\eta\rho^*\zeta_{\alpha^*}]^*[\rho'\zeta_\alpha+\eta'\rho'^*\zeta_{\alpha^*}]$$
$$+[\rho\zeta_\alpha-\eta\rho^*\zeta_{\alpha^*}]^*[\rho'\zeta_\alpha-\eta'\rho'^*\zeta_{\alpha^*}])$$
$$=2Y_{\gamma\delta\mu\nu}(\rho^*\rho'\zeta_\alpha{}^*\zeta_\alpha+\eta\eta'\rho\rho'^*\zeta_{\alpha^*}{}^*\zeta_{\alpha^*})\,.$$

Since ζ_α is proportional to $\exp[\alpha(u)\ln s]$, one thus has that $\zeta_\alpha\zeta_\alpha{}^*=\zeta_{\alpha^*}{}^*\zeta_{\alpha^*}$ is proportional to $s^{2\,\mathrm{Re}\alpha(u)}$. Thus (independent of all the exact details of the above polarization expressions) there will be no oscillatory terms, leading order in s, in any final-state polarizations when the initial state is unpolarized. This is in agreement with the findings of Uberall and of Gribov, Okun, and Pomeranchuk[4] for πN scattering.

When the initial state is polarized, the α, β sums now

[13] Interference between different fermion trajectories can still give an oscillating term; but this will be smaller by $S^{2\,\mathrm{Re}(J_1-J_2)}$.

[14] The author thanks Dr. H. Uberall for a remark about a misimpression of the author in a preliminary preprint; his remark instigated the work of this section.

[15] See Williams, *An Introduction to Elementary Particles* (Academic Press Inc., New York, 1961).

will involve tensorially contracted products with rotation matrices and with an initial-state density matrix, and the above results are not expected to obtain.

It is simple to see,[4] also, that if two close-lying pairs of trajectories are present, both polarization and $d\sigma/d\Omega$ can exhibit oscillations, but these will be down by a factor s to the power $(\alpha_2-\alpha_1)$ from the dominant terms in the relevant expressions. (Oscillations have been shown to occur, in this situation, in π-N scattering, by Gribov *et al.*[4])

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Unitary Triplets and the Eightfold Way*

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In order to explain the eightfold way, four elementary baryon fields are introduced. Three of them form a unitary triplet and the fourth is a unitary singlet. In this approach, triplets, sextets, etc., are possible multiplets as well as singlets, octets, decuplets, etc. This model has a new quantum number, "hypercharge center." Assuming that the symmetry-breaking interactions transform like components of a triplet, selection rules in the production and decay of the triplets are derived. It is proposed that the isodoublet $\kappa(725)$ along with the isosinglet $z^+(Y=2)$ or $\eta'(Y=0)$ forms a unitary triplet. If the symmetry-breaking interaction transforms like a component of the octet, the following baryon lepton symmetry suggested by Gell-Mann

$$\nu_e \leftrightarrow \text{"}p\text{"}\cos\theta+\text{"}Z\text{"}\sin\theta,$$
$$\nu_\mu \leftrightarrow -\text{"}p\text{"}\sin\theta+\text{"}Z\text{"}\cos\theta$$
$$e^- \leftrightarrow \text{"}n\text{"}\cos\theta'+\text{"}\Lambda'\text{"}\sin\theta',$$
$$\mu^- \leftrightarrow -\text{"}n\text{"}\sin\theta'+\text{"}\Lambda'\text{"}\cos\theta',$$

between four leptons and four elementary baryon fields is shown to be possible.

I. INTRODUCTION

THE success of the broken eightfold way is striking.[1,2] Some have looked for its origin in the bootstrap mechanism.[3,4] However, it is not easy to understand why the bootstrap mechanism prefers the octet scheme of the SU(3) symmetry to other models. The origin of an internal symmetry is most easily understood by introducing elementary fields and a symmetric Lagrangian.

In order to explain the eightfold way, at least four elementary fields are necessary. If we assume the elementary fields are singly charged or neutral, the following two possibilities exist[5]: (a) "p," "n," "Λ," and "Λ'" are elementary where "p" ($B=1$, $Y=1$, $I=\frac{1}{2}$, $Q=1$), "n" ($B=1$, $Y=1$, $I=\frac{1}{2}$, $Q=0$), and "Λ" ($B=1$, $Y=0$, $I=0$, $Q=0$) form a unitary triplet which transforms like 3, and where "Λ'" ($B=1$, $Y=0$, $I=0$, $Q=0$) is a unitary singlet.[6] (b) "n," "p," "Z," and "Λ'" are elementary, where "n," "p," and "Z" ($B=1$, $Y=2$, $I=0$, $Q=1$) form a unitary triplet which transforms like 3*, and where "Λ'" is a unitary singlet.[7]

Here, the fields "p," "n," and "Λ" have no relation to the real p, n, and Λ, which are components of a unitary octet, except that they have the same baryon number, hypercharge, isotopic spin, and charge. At the present time, the particles associated with the fields "p," "n," "Λ," "Z," and "Λ'" have not yet been observed. Their masses must be very large—they may even be infinite.

In this model all stable and unstable particles are considered to be bound states of these elementary particles, and they belong to multiplets corresponding to irreducible representations of SU(3) symmetry. In this article, the mechanism of their dynamical emergence will not be discussed, however.

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[6] In the following, the symbols B_1, B_2, B_3, and B are used for "p," "n," "Λ," and "Λ'," respectively, for case (a).
[7] In the following, the symbols B^1, B^2, B^3, and B are used for "n," "p," "Z," and "Λ'," respectively, for case (b).